

The CAPM, APT and Multifactor Models of Risk and Return

Topic 5
Chapters 9&10, Bodie, Kane&Marcus
Lectured by Jing Chi, Ph.D., CFA



Main Points

- The capital asset pricing model (CAPM)
- The CAPM and the index model
- APT
- Multifactor models



Capital Asset Pricing Model (CAPM)

- Derived using principles of diversification with simplified assumptions
- Showing the relationship between risk and expected return of an asset
- Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development



Assumptions

- Individual investors are price takers
- Single-period investment horizon (ignoring anything that might happen afterwards)
- Investments are limited to traded financial assets (can borrow or lend at risk-free rate)

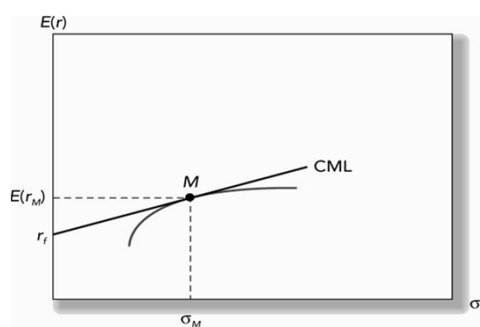
Assumptions

- No taxes and transaction costs
- Investors are rational mean-variance optimizers
- There are homogeneous expectations
- Information is costless and available to all investors

Resulting Equilibrium Conditions

- All investors will hold the same portfolio for the optimal risky assets – market portfolio, differing only in the amount invested in it vs. in the risk-free asset; CAL becomes CML
- Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value

Capital Market Line



Resulting Equilibrium Conditions

- Risk premium on the market portfolio will be proportional to its risk and the average risk aversion of all market participants

$$E(r_M) - r_f = \bar{A} \sigma_M^2 * 0.01$$

- Since M is the optimal portfolio (diversified across all assets), the variance of the market portfolio is the systematic risk of this universe

Resulting Equilibrium Conditions

- Risk premium on an individual security will be proportional to the risk premium on the market portfolio and the beta coefficient of the security relative to the market portfolio (beta measures the extent to which returns on the stock and the market move together)

$$\beta_i = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}$$

$$E(r_i) - r_f = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2} [E(r_M) - r_f] = \beta_i [E(r_M) - r_f]$$

CAPM

- CAPM assumes that appropriate risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios
- Investors have the same input list (expected returns, S.D. & covariance)

CAPM

- Covariance matrix

weight	W1	W2	W _{GM}	...Wn
W1	Cov(r1,r1)	Cov(r1,r2)	Cov(r1,r _{GM})	Cov(r1,rn)
W2	Cov(r2,r1)	Cov(r2,r2)	Cov(r2,r _{GM})	Cov(r2,rn)
W _{GM}	Cov(r _{GM} ,r1)	Cov(r _{GM} ,r2)	Cov(r _{GM} ,r _{GM})	Cov(r _{GM} ,rn)
...	...	...	...	...
Wn	Cov(rn,r1)	Cov(rn,r2)	Cov(rn,r _{GM})	Cov(rn,rn)

CAPM

- The variance of the portfolio is sum of all elements
- The contribution of GM's stock to the variance of the market portfolio is

$$W_{GM} [W_1 \text{Cov}(r_1, r_{GM}) + W_2 \text{Cov}(r_2, r_{GM}) + \dots + W_{GM} \text{Cov}(r_{GM}, r_{GM}) + \dots + W_n \text{Cov}(r_n, r_{GM})]$$
- GM's contribution to variance =

$$W_{GM} \text{Cov}(r_{GM}, r_M)$$

CAPM

- The contribution of our holding of GM to the risk premium of the market portfolio is $W_{GM}[E(r_M) - r_f]$
- The reward-to-risk ratio for GM is
GM's contribution to risk premium / GM's contribution to variance = $W_{GM}[E(r_{GM}) - r_f] / W_{GM} \text{Cov}(r_{GM}, r_M)$

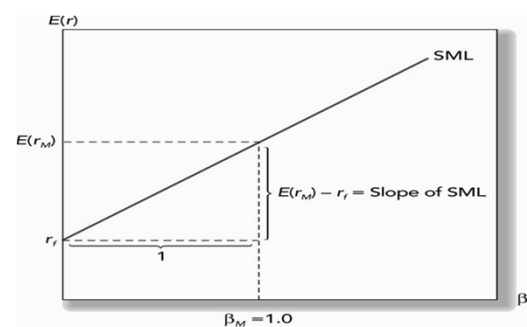
CAPM

- The reward-to-risk ratio for market portfolio is
Market risk premium / Market variance
= $[E(r_M) - r_f] / \sigma_M^2$
- A basic principle of equilibrium is that all investments should offer the same reward-to-risk ratio, so
 $[E(r_{GM}) - r_f] / \text{Cov}(r_{GM}, r_M) = [E(r_M) - r_f] / \sigma_M^2$
 $E(r_{GM}) - r_f = \frac{\text{Cov}(r_{GM}, r_M)}{\sigma_M^2} [E(r_M) - r_f]$
 $E(r_{GM}) = r_f + \beta_{GM} [E(r_M) - r_f]$

CAPM

- CAPM
 $E(r_{GM}) = r_f + \beta_{GM} [E(r_M) - r_f]$
- The risk premium on individual assets depends on the contribution of the individual asset to the risk of the portfolio
- Beta measures the stock's contribution to the variance of the market portfolio

Security Market Line



SML Relationships

$$\beta = [\text{COV}(r_i, r_m)] / \sigma_m^2$$

$$\text{Slope SML} = E(r_m) - r_f$$

$$= \text{market risk premium}$$

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f]$$

$$\text{Beta}_m = [\text{Cov}(r_m, r_m)] / \sigma_m^2$$

$$= \sigma_m^2 / \sigma_m^2 = 1$$

SML and CML

- CML graphs the risk premiums of efficient portfolios as a function of portfolio standard deviation
- SML graphs individual asset risk premiums as a function of asset risk; the measure of risk for individual assets held as parts of well-diversified portfolios is the contribution of the asset to the portfolio variance (beta)

Sample Calculations for SML

$$E(r_m) - r_f = .08 \quad r_f = .03$$

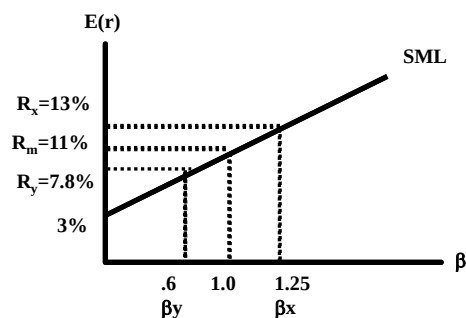
$$\beta_x = 1.25$$

$$E(r_x) = .03 + 1.25(.08) = .13 \text{ or } 13\%$$

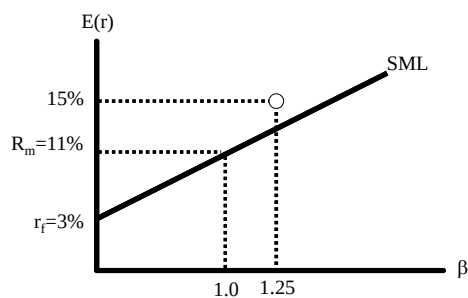
$$\beta_y = .6$$

$$E(r_y) = .03 + .6(.08) = .078 \text{ or } 7.8\%$$

Graph of Sample Calculations



Disequilibrium Example



Disequilibrium Example (cont.)

- Suppose a security with a β of 1.25 is offering expected return of 15%
- According to SML, it should be 13%
- Under-priced: offering too high of a rate of return for its level of risk (positive alphas)
- You should buy

Using CAPM

- CAPM can be used to analyze security; portfolio manager will increase the weights of securities with positive alphas and decrease the weights of securities with negative alphas
- CAPM is useful in asset or project valuation by providing required rate of return (NPV valuation; DDM valuation)

The CAPM and Reality

- Is the condition of zero alphas for all stocks as implied by the CAPM met?
 - Not perfect but one of the best available
- Is the CAPM testable?
 - Proxies must be used for the market portfolio
- CAPM is still considered one of the best available description of security pricing and is widely accepted

Single Index Model & CAPM

- What's the difference between beta in CAPM and beta in Single Index Model?
- According to single index model, the covariance between stock 'i' and the market is

$$\begin{aligned} \text{Cov}(R_i, R_M) &= \beta_i \beta_M \sigma_M^2 = \beta_i \sigma_M^2 \\ \beta_i &= \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} \end{aligned}$$

- They look the same, however, CAPM measures expected returns, while single index model measures real/historical returns

Single Index Model & CAPM

- CAPM

$$E(r_i) - r_f = \beta_i [E(r_M) - r_f]$$

- Single Index Model

$$E(r_i) - r_f = \alpha_i + \beta_i [E(r_M) - r_f]$$

- The CAPM state that the expected value of alpha is zero for all securities, whereas the index model holds that the realized value of alpha should average out to zero for a sample of historical observed returns

Single Index Model & CAPM

- "M" in two models are different
 - In single index model, "m" stands for a market index
 - In CAPM, "m" stands for the whole market, which consists of all securities in the market

Arbitrage Pricing Theory (APT)

Arbitrage - arises if an investor can construct a zero investment portfolio with a sure profit

- Since no investment is required, an investor can create large positions to secure large levels of profit
- In efficient markets, profitable arbitrage opportunities will quickly disappear

Arbitrage Pricing Theory

- More general than the CAPM, with less restrictive assumptions.
- Unlike CAPM, APT does not assume:
 - Single-period investment horizon
 - Absence of personal taxes
 - Risk-free borrowing or lending
 - Mean-variance decisions

APT Model

- What is most important in this model are the deviations of the factors from their expected values.
- The expected return-risk relationship for the APT can be described as:

$$R_i = E(R_i) + \beta_{i1}f_1 + \beta_{i2}f_2 + \dots + \beta_{in}f_n + e_i$$

APT and CAPM Compared

- APT applies to well diversified portfolios and not necessarily to individual stocks
- APT is more general in that it gets to an expected return and beta relationship without the assumption of the market portfolio
- APT can be extended to multifactor models

Multifactor Models

- Use more than one factor that are important to performance of the general economy in addition to market return
 - Examples include gross domestic product, interest rates etc.
 - Estimate a beta for each factor using multiple regression

Multifactor Model Equation

$$R_i = E(r_i) + \text{Beta}_{\text{GDP}} (\text{GDP}) + \text{Beta}_{\text{IR}} (\text{IR}) + e_i$$

R_i = Return for security i

Beta_{GDP} = Factor sensitivity for GDP

Beta_{IR} = Factor sensitivity for interest rate

e_i = Firm specific events

Fama-French Three-Factor Model

□ The factors chosen are variables that on past evidence seem to predict average returns well and may capture the risk premiums

$$r_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_{it}$$

Fama-French Three-Factor Model

- SMB = Small Minus Big, i.e., the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks
- HML = High Minus Low, i.e., the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio

Example--1

- The security market line depicts:
 - a. A security's expected return as a function of its systematic risk
 - b. The market portfolio as the optimal portfolio of risky securities
 - c. The relationship between a security's return and the return on an index
 - d. The complete portfolio as a combination of the market portfolio and the risk-free asset

Example--2

- Within the context of the CAPM, assume:

$E(r_M) = 15\%$; $r_f = 8\%$; $E(r_{XYZ}) = 17\%$;

$Beta_{(XYZ)} = 1.25$

Which one is correct?

- a. XYZ stock is overpriced
- b. XYZ stock is fairly priced
- c. XYZ stock's alpha is $-.25\%$
- d. XYZ stock's alpha is $.25\%$

Example--3

- According to CAPM, the expected rate of return of a portfolio with a beta of 1.0 and an alpha of 0 is:

- a. Between r_M and r_f
- b. The risk-free rate
- c. $Beta \cdot (r_M - r_f)$
- d. The expected return on the market, r_M

Example--4

- Beta and standard deviation differ as risk measures in that beta measures:

- a. Only unsystematic risk, while S.D. measures total risk
- b. Only systematic risk, while S.D. measures total risk
- c. Both systematic and unsystematic risk, while S.D. measures only unsystematic risk
- d. Both systematic and unsystematic risk, while S.D. measures only systematic risk

Example--5

- The correlation between Fund A and the market index is 1.0. The expected return on the market index is 11%, the expected return on Fund A is 9%, and the risk-free return is 3%. Based on this analysis, the implied beta of Fund A is:

- a. Negative
- b. 0.75
- c. 0.82
- d. 1.00